**Logic Puzzles**

### The Game- Prisoner’s Dilemma

Tucker began with a little story, like this: two burglars, Bob and Al, are captured near the scene of a burglary and are given the "third degree" separately by the police. Each has to choose whether or not to confess and implicate the other. If neither man confesses, then both will serve one year on a charge of carrying a concealed weapon. If each confesses and implicates the other, both will go to prison for 10 years. However, if one burglar confesses and implicates the other, and the other burglar does not confess, the one who has collaborated with the police will go free, while the other burglar will go to prison for 20 years on the maximum charge.

The strategies in this case are: confess or don't confess. The payoffs (penalties, actually) are the sentences served. We can express all this compactly in a "payoff table" of a kind that has become pretty standard in game theory. Here is the payoff table for the Prisoners' Dilemma game:

**Table 3-1**

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | Al | |
|  |  | confess | don't |
| Bob | confess | 10,10 | 0,20 |
| don't | 20,0 | 1,1 |

The table is read like this: Each prisoner chooses one of the two strategies. In effect, Al chooses a column and Bob chooses a row. The two numbers in each cell tell the outcomes for the two prisoners when the corresponding pair of strategies is chosen. The number to the left of the comma tells the payoff to the person who chooses the rows (Bob) while the number to the right of the column tells the payoff to the person who chooses the columns (Al). Thus (reading down the first column) if they both confess, each gets 10 years, but if Al confesses and Bob does not, Bob gets 20 and Al goes free.

So: how to solve this game? What strategies are "rational" if both men want to minimize the time they spend in jail? Al might reason as follows: "Two things can happen: Bob can confess or Bob can keep quiet. Suppose Bob confesses. Then I get 20 years if I don't confess, 10 years if I do, so in that case it's best to confess. On the other hand, if Bob doesn't confess, and I don't either, I get a year; but in that case, if I confess, I can go free. Either way, it's best if I confess. Therefore, I'll confess."

But Bob can and presumably will reason in the same way -- so that they both confess and go to prison for 10 years each. Yet, if they had acted "irrationally," and kept quiet, they each could have gotten off with one year each.

**The Sorites Paradox:**

## The name ‘Sorites’ derives from the Greek word soros (meaning ‘heap’) and originally referred, not to a paradox, but rather to a puzzle known as The Heap: Would you describe a single grain of wheat as a heap? No. Would you describe two grains of wheat as a heap? No. … You must admit the presence of a heap sooner or later, so where do you draw the line?

It was one of a series of puzzles attributed to the Megarian logician Eubulides of Miletus. Also included were:

The Liar: A man says that he is lying. Is what he says true or false?

The Hooded Man: You say that you know your brother. Yet that man who just came in with his head covered is your brother and you did not know him.

The Bald Man: Would you describe a man with one hair on his head as bald? Yes. Would you describe a man with two hairs on his head as bald? Yes. … You must refrain from describing a man with ten thousand hairs on his head as bald, so where do you draw the line?

This last puzzle, presented as a series of questions about the application of the predicate ‘is bald’, was originally known as the falakros puzzle. It was seen to have the same form as the Heap and all such puzzles became collectively known as sorties puzzles.

It is not known whether Eubulides actually invented the sorties puzzles. Some scholars have attempted to trace its origins back to Zeno of Elea but the evidence seems to point to Eubulides as the first to employ the sorties. Nor is it known just what motives Eubulides may have had for presenting this puzzle. It was, however, employed by later Greek philosophers to attack various positions, most notably by the Skeptics against the Stoics' claims to knowledge.

These puzzles of antiquity are now more usually described as paradoxes. Though the conundrum can be presented informally as a series of questions whose puzzling nature gives it dialectical force it can be, and was, presented as a formal argument having logical structure. The following argument form of the sorties was common:

1 grain of wheat does not make a heap.   
If 1 grain of wheat does not make a heap then 2 grains of wheat do not.   
If 2 grains of wheat do not make a heap then 3 grains do not.   
…   
If 9,999 grains of wheat do not make a heap then 10,000 do not.   
--------------------------------------------------------------------   
10,000 grains of wheat do not make a heap.

The argument certainly seems to be valid, employing only modus ponens and cut (enabling the chaining together of each sub-argument which results from a single application of modus ponens). These rules of inference are endorsed by both Stoic logic and modern classical logic, amongst others.

Moreover, its premises appear true. Some Stoic presentations of the argument recast it in a form which replaced all the conditionals, ‘If A then B’, with ‘Not (A and not-B)’ to stress that the conditional should not be thought of as being a strong one, but rather the weak Philonian conditional (the modern material conditional) according to which ‘If A then B’ was equivalent to ‘Not (A and not-B)’. Such emphasis was deemed necessary since there was a great deal of debate in Stoic logic regarding the correct analysis for the conditional. In thus judging that a connective as weak as the Philonian conditional underpinned this form of the paradox they were forestalling resolutions of the paradox that denied the truth of the conditionals based on a strong reading of them. This interpretation then presents the argument in its strongest form since the validity of modus ponens seems assured whilst the premises are construed so weakly as to be difficult to deny. The difference of one grain would seem to be too small to make any difference to the application of the predicate; it is a difference so negligible as to make no apparent difference to the truth-values of the respective antecedents and consequents.

Yet the conclusion seems false. Thus, paradox confronted the Stoics just as it does the modern classical logician. Nor are such paradoxes isolated conundrums. Innumerable sorties paradoxes can be expressed in this way. For example, one can present the puzzle of the Bald Man in this manner. Since a man with one hair on his head is bald and if a man with one is then a man with two is, so a man with two hairs on his head is bald. Again, if a man with two is then a man with three is, so a man with three hairs on his head is bald, and so on. So, a man with ten thousand hairs on his head is bald, yet we rightly feel that such men are hirsute, i.e., not bald. Indeed, it seems that almost any vague predicate admits of such a sorties paradox and vague predicates are ubiquitous.

As presented, the paradox of the Heap and the Bald Man proceed by addition (of grains of wheat and hairs on the head respectively). Alternatively, though, one might proceed in reverse, by subtraction. If one is prepared to admit that ten thousand grains of sand make a heap then one can argue that one grain of sand does since the removal of any one grain of sand cannot make the difference. Similarly, if one is prepared to admit a man with ten thousand hairs on his head is not bald, then one can argue that even with one hair on his head he is not bald since the removal of any one hair from the originally hirsute scalp cannot make the relevant difference. It was thus recognized, even in antiquity, that sorties arguments come in pairs, using: ‘non-heap’ and ‘heap’; ‘bald’ and ‘hirsute’; ‘poor’ and ‘rich’; ‘few’ and ‘many’; ‘small’ and ‘large’; and so on. For every argument which proceeds by addition there is another reverse argument which proceeds by subtraction.

Curiously, the paradox seemed to attract little subsequent interest until the late nineteenth century when formal logic once again assumed a central role in philosophy. Since the demise of ideal language doctrines in the latter half of the twentieth century interest in the vagaries of natural language and the sorties paradox in particular, has greatly increased.

**Buridan’s Donkey:**

A donkey is positioned exactly between a pile of hay and a watering trough. Logic dictates that the donkey will go to the need that is closer to him. Therefore, the donkey dies.

-Is there such a thing as free will?

- What would be the logical choice for the donkey?

- What are some real-world applications of this principle?